# $M_{W}$ and $\sin ^{2} \theta_{\text {eff }}$ in split SUSY: present and future expectations 

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AbStract: We analyse the precision electroweak observables $M_{W}$ and $\sin ^{2} \theta_{\text {eff }}$ and their correlations in the recently proposed Split SUSY model. We compare the results with the Standard Model and Minimal Supersymmetric Standard Model predictions, and with present and future experimental accuracies. Present experimental accuracies in $\left(M_{W}\right.$, $\sin ^{2} \theta_{\text {eff }}$ ) do not allow constraints to be placed on the Split SUSY parameter space. We find that the shifts in $\left(M_{W}, \sin ^{2} \theta_{\text {eff }}\right)$ induced by Split SUSY can be larger than the anticipated accuracy of the GigaZ option of the International Linear Collider, and that the most sensitive observable is $\sin ^{2} \theta_{\text {eff }}$. These large shifts are possible also for large chargino masses in scenarios with small $\tan \beta \simeq 1$.

Keywords: Quark Masses and SM Parameters, Supersymmetry Phenomenology.

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## 1. Introduction

The Standard Model (SM) with minimal Higgs-field content could turn out not to be the basic theoretical framework for describing electroweak symmetry breaking. During the last decades, Supersymmetry (SUSY) has become one of the most promising theoretical ideas beyond the SM. The Minimal Supersymmetric Standard Model (MSSM) [1, 2] is the simplest supersymmetric extension of the SM, and it is at least as successful as the SM to describe the experimental data [3]. This model predicts the existence of scalar partners $\tilde{f}_{\mathrm{L}}$, $\tilde{f}_{\mathrm{R}}$ to each SM chiral fermion, and of spin- $1 / 2$ partners to the gauge and Higgs bosons. It is found that the effects of SUSY at the scale of $\mathcal{O}(\mathrm{TeV})$ can provide a theoretically well motivated solution to the hierarchy problem and also predicts the unification of the gauge couplings [ [4, 呵]. Moreover, the lightest neutralino in SUSY models constitutes a promising dark matter candidate [4]. However, in spite of the above successes, SUSY still has some unsolved problems for phenomenological reasons. For instance, large flavour mixing and proton decay, as well as a too large cosmological constant, are predicted by these models.

Recently, the scenario of Split SUSY has been suggested [6-8]. In this scenario, the SUSY-breaking scale is much heavier than the electroweak scale, i.e. there is a hierarchy between the scalar superpartners and the fermionic partners of the SM particles. Therefore, except for one Higgs-boson, all scalar particles (squarks, sleptons and extra MSSM Higgs particles) are very heavy, of the order of $10^{9} \mathrm{GeV}$, while the fermions (gauginos and higgsinos) are kept at the electroweak scale. Thus, only the SM spectrum, including one Higgs scalar, and gauginos and higgsinos remain. The rest of the MSSM spectrum decouples [9, 10]. This scenario implies the existence of an "unnatural" fine-tuning, such that the Higgs-boson vacuum expectation value can be kept at the observed electroweak scale. Assuming this fine-tuning effect, some of the remaining problems in SUSY models are solved: as a consequence of decoupling of all sfermions, there is no flavour-changing neutral current problem that emerges in the MSSM, and the mediating proton decay problem has been eliminated. On the other hand, keeping gauginos and higgsinos at the electroweak scale, gauge unification is preserved and we can have a neutralino as a good candidate for dark matter.

Phenomenological implications of Split SUSY have been extensively discussed during the last year [11]. An alternative way, with respect to the direct search for beyond the SM physics or Higgs particles, is to probe new physics through virtual effects of the additional non-standard particles to precision observables. In particular, the analysis of radiative effects of light gauginos and higgsinos to precision electroweak (EW) observables in Split SUSY have been presented in [12]. The analysis used the $S, T, U$ parameter expansions, as well as corrections from non-zero momentum summarized in $Y, V, W$ parameters [13]15]. They found that the precision electroweak data are compatible with the Split SUSY spectrum for the values of gaugino and higgsino masses above the direct collider limits. Moreover, Split SUSY corrections to precision observables after LEP2, and by considering also the contributions of LEP1 only, are studied in [16]. For LEP2, the SM prediction fits better than Split SUSY predictions, but the difference between the two fits is not "spectacular". For the LEP1 analysis, on the contrary, the description of the data fits better in Split SUSY than in the SM (but not dramatically).

The analysis of virtual effects of the additional non-standard particles on new physics models to precision observables requires a very high precision of the experimental results as well as of the theoretical predictions. A predominant role in this respect has to be assigned to the $\rho$-parameter [17], with loop contributions $\Delta \rho$ through vector-boson self-energies, which constitute the leading process-independent quantum corrections to electroweak precision observables, such as the prediction for $\Delta r$, the $M_{W}-M_{Z}$ interdependence, and the effective leptonic weak mixing angle, $\sin ^{2} \theta_{\text {eff }}$. Radiative corrections to the electroweak precision observables within the MSSM have been extensively discussed (for a review see, e.g. (3]). In particular, a detailed analysis of the SM and the MSSM predictions in the $M_{W}-\sin ^{2} \theta_{\text {eff }}$ plane, by considering the prospective accuracies for the Large Hadron Collider (LHC) and the International Linear Collier (ILC) with GigaZ option, is included in 18, 19. The authors found that the MSSM is slightly favoured over the SM, depending of the central value of the experimental data. Once the $W$ mass, the effective leptonic weak mixing angle, $\sin ^{2} \theta_{\text {eff }}$, as well as the top-quark mass, crucial in this analysis, become known with better accuracy at future colliders, a very high precision of the theoretical predictions for these observables from both SM and new physics is needed.

Now we study the effects of gauginos and higgsinos on the $M_{W}-\sin ^{2} \theta_{\text {eff }}$ interdependence in Split SUSY, i.e. when the scalar superpartner masses are too heavy. We focus on the comparison of Split SUSY predictions with the SM and MSSM predictions, by considering the present data and the prospective experimental precision at the next generation of colliders. Even if the regions of parameter space allowed by colliders constraints is expected to be allowed by precision electroweak constraints in Split SUSY 12, 16, an analysis of these two observables, which are very precisely determined by experiments [20, 21], has not yet been done, and could provide extra information about the compatibility and/or similarities and differences between Split SUSY predictions on these two EW precision observables and the SM and MSSM predictions. A few more words are in order with respect the recent analysis in refs. [12, 16]. The authors of refs. [12, 16] focus on the analysis of current experimental data, performing a $\chi^{2}$ fit, and finding whether Split SUSY fits better current experimental data than the SM. Our work focusses on the possibility of detecting
the deviations induced by Split SUSY in the present and future measurements of $M_{W}$ and $\sin ^{2} \theta_{\text {eff }}$.

Since the Higgs-boson enters the two electroweak precision observables we are interested in (by virtue of its contributions to the self-energies of electroweak vector bosons) an analysis of the radiative corrections to the Higgs scalar boson mass from Split SUSY must be included in our study. It is already known that the strong constraints on the parameters of low-energy SUSY imposed by the lower bound on the Higgs-boson mass, $m_{H}>114.4 \mathrm{GeV}$ [22], are relaxed in Split SUSY. This is thanks to the large corrections to this mass, due to the renormalization group evolution from the scale of heavy scalars to the weak scale [7]. These effects have been taken into account in our analysis by using the renormalization group evolution as given in ref. [7].

## 2. $M_{W}$ and $\sin ^{2} \theta_{\text {eff }}$ electroweak precision observables

Precisely measured observables such as the $W$-boson mass, $M_{W}$, and the effective leptonic mixing angle, $\sin ^{2} \theta_{\text {eff }}$, are affected by shifts according to

$$
\begin{equation*}
\delta M_{W} \approx \frac{M_{W}}{2} \frac{\cos ^{2} \theta_{W}}{\cos ^{2} \theta_{W}-\sin ^{2} \theta_{W}} \Delta \rho, \quad \delta \sin ^{2} \theta_{\mathrm{eff}} \approx-\frac{\cos ^{2} \theta_{W} \sin ^{2} \theta_{W}}{\cos ^{2} \theta_{W}-\sin ^{2} \theta_{W}} \Delta \rho \tag{2.1}
\end{equation*}
$$

$\theta_{W}$ being the weak mixing angle, and the electroweak $\rho$ parameter given by $\Delta \rho=\frac{\Sigma_{Z}(0)}{M_{Z}^{2}}-$ $\frac{\Sigma_{W}(0)}{M_{W}^{2}}$, with $\Sigma_{Z, W}(0)$ the unrenormalized $Z$ and $W$ boson self-energies at zero momentum. We remark that, beyond the $\Delta \rho$ approximation, the shifts in these two observables, entering through self-energy corrections, are given in terms of the $\delta(\Delta r)$ quantity. However, the computation and discussion of contributions to $\Delta r$ in Split SUSY reduces to the corresponding analysis of the $\Delta \rho$ quantity. In general $\Delta r$ is given in terms of the photon vacuum polarization, the ratio of the strengths of neutral and charged currents at vanishing momentum transfer $(\Delta \rho)$, and the remainder vertex and boxes contributions. However, if we are interested in extra contributions to $\Delta r$ that are not in the SM, each non-SM contribution to the 3 - and 4 -point functions contains at least one scalar particle. This scalar can be either the lightest Higgs-boson and, therefore, is like an SM contribution; or a heavy Higgs-boson or a slepton, whose contribution is negligible, since these scalar particles have a mass of $\mathcal{O}\left(10^{9} \mathrm{GeV}\right)$ in Split SUSY. As a consequence, no extra contribution to $\Delta r$ emerges in this model, and the analysis can be reduced to the computation of $\Delta \rho$ contributions.

For our computation, we have used ZFITTER [23, 24] for the SM prediction. The MSSM contributions to $\Delta r$ have been taken from ref. 25-28], and we have used FeynArts/FormCalc/LoopTools [29-34] for the vertex contributions to $\sin ^{2} \theta_{\text {eff }}$. The Higgs-boson mass is computed according to ref. $\sqrt[7]{7}$ for Split SUSY, and using the leading $m_{t}, m_{b} \tan \beta$ approximation for the MSSM [35-38]. The Split SUSY/MSSM contributions to $\Delta r$ are added to the ZFITTER computation, and we proceed in an iterative way to compute $M_{W}, \sin ^{2} \theta_{W}$. As for the input parameters, we have used $M_{Z}=91.1876 \mathrm{GeV}, \alpha^{-1}(0)=137.0359895$ [39], $\Delta \alpha_{\text {had }}^{5}\left(M_{Z}\right)=0.02761 \pm 0.00036$ (40] (corresponding to $\alpha^{-1}\left(M_{Z}\right)=128.936$ ), $\alpha_{s}\left(M_{Z}\right)=$
$0.119 \pm 0.003$ [40]. For the top-quark mass, we use the latest combination of RunI/II Tevatron data: $m_{t}=172.7 \pm 2.9 \mathrm{GeV}$ [41].

The parameter space of Split SUSY is formed by the higgsino mass parameter $\mu$, the electroweak gaugino soft-SUSY-breaking mass parameters $M_{1}$ and $M_{2}$ (we use the GUT mass relation $M_{1}=M_{2} 5 / 3 \tan ^{2} \theta_{W}$ ), the gluino soft-SUSY-breaking mass $M_{g}$, the ratio between the vacuum expectation values of the two Higgs doublets $\tan \beta=v_{2} / v_{1}$, and the scale of the scalar particles masses $\tilde{m}$. The most important phenomenological consequence of Split SUSY is the presence of a long-lived gluino [6, (7, 42-45]. The scalar mass scale ( $\tilde{m}$ ) lays between the EW scale ( $\sim 1 \mathrm{TeV}$ ) and the unification scale ( $\sim 10^{16} \mathrm{GeV}$ ), current limits from gluino cosmology set an upper bound $\tilde{m} \lesssim 10^{9} \mathrm{GeV}$ 455. In our computation the gluino mass $\left(M_{g}\right)$ and the scalar scale ( $\tilde{m}$ ) enter the Higgs-boson mass computation, the latter defining the matching scale with the SUSY theory, and the former through the running of the top quark Yukawa coupling. For definiteness, we will use $\tilde{m}=10^{9} \mathrm{GeV}$, while $M_{g}$ is let free.

## 3. Results

Now we focus on the comparison for $M_{W}$ and $\sin ^{2} \theta_{\text {eff }}$ predictions from different models with the present data and the prospective experimental precision at the next generation of colliders. The results for the SM, the MSSM and Split SUSY predictions are given in figure 1 , in the $M_{W}-\sin ^{2} \theta_{\text {eff }}$ plane. The top-quark mass is varied in the $3 \sigma$ range of the current experimental determination. Predictions are shown together with the experimental results for $M_{W}$ and $\sin ^{2} \theta_{\text {eff }}$ (using the current central values: $M_{W}=80.410 \pm 0.032 \mathrm{GeV}, \sin ^{2} \theta_{\text {eff }}=0.231525 \pm 0.00016$ ) and the prospective accuracies at present (LEP2/SLD/Tevatron) and at the next generations of colliders (LHC/ILC and the GigaZ option) [20, 21]. Our results agree with previous ones for the SM and the MSSM predictions given in 18, 19.

First, we concentrate on results given in figure 1a. We have performed a Monte Carlo scan of the respective parameter space of the different models, taking into account current experimental limits on new particles, to find the allowed region in the $M_{W}-\sin ^{2} \theta_{\text {eff }}$ plane for each model. The allowed regions are those enclosed by the different curves. The arrows show the direction of change in these regions as the given parameters grow. The shaded region corresponds to the SM prediction, and it arises from varying the mass of the SM Higgs-boson, from 114 GeV [22] to 400 GeV . The region enclosed by the dash-dotted curve corresponds to the MSSM. Here the SUSY masses are varied between 2 TeV (corresponding to the upper edge of the area) and close to their experimental lower limit $m_{\chi} \gtrsim 100 \mathrm{GeV}$, $m_{\tilde{f}} \gtrsim 150 \mathrm{GeV}$ (lower edge of the band). As is very well known, contrary to the SM case, the lightest MSSM Higgs-boson mass is not a free parameter. Thus, the overlap region between SM and MSSM corresponds to the region where the Higgs-boson is light, i.e. in the MSSM allowed region $m_{h^{0}}<140 \mathrm{GeV}$ [3], all superpartners being heavy (decoupling limit in the MSSM), as already established in [18, 19]. The Split SUSY prediction is summarized in the region enclosed by the black line in this figure. Here, the scalar particles masses are of the order of $10^{9} \mathrm{GeV}$, and the Higgs-boson mass is computed by following the equations of the


Figure 1: SM, MSSM and Split SUSY predictions for $M_{W}$ and $\sin ^{2} \theta_{\text {eff }}$. The ellipses are the experimental results for $M_{W}$ and $\sin ^{2} \theta_{\text {eff }}$ and the prospective accuracies at LEP2/SLD/Tevatron (large ellipse), LHC/ILC (medium ellipse) and GigaZ (small ellipse).
renormalization group evolution as in (7). The computed Higgs-boson mass varies in the range $m_{H}^{\text {split }} \sim 110-153 \mathrm{GeV}$. The region excluded by the experimental Higgs-boson mass limit $m_{H} \lesssim 114 \mathrm{GeV}$ [22] corresponds to a tiny corner of the parameter space: $\tan \beta<1.5$, $m_{t}<166 \mathrm{GeV}$. As expected, we found overlap regions between Split SUSY and both the SM and the MSSM. Moreover, we see that most of the region predicted by Split SUSY for $M_{W}$ and $\sin ^{2} \theta_{\text {eff }}$ overlaps with predictions already given by the SM and the MSSM.

In order to clarify the differences of predictions induced by the three models, we focus on the analysis of the region in which they overlap. The corresponding results are shown in figure 1 b (notice the different scales of the two plots in this figure). Here the SM prediction (shaded area) is fixed to be the one obtained when the SM Higgs-boson mass is varied in the range of the Split SUSY prediction $m_{H}=114-153 \mathrm{GeV}$. It allows the extraction of the exact overlap region between SM and Split SUSY predictions, by assuming the same Higgs-boson mass value in the two models. The MSSM results remain as before. The region in which the MSSM and Split SUSY overlap corresponds to having heavy scalar particles (decoupling of squarks, sleptons and extra Higgs bosons in the MSSM) and a Higgs-boson mass around 140 GeV (upper edge of the dash-dotted area). However, this small region that does not exist in the MSSM emerges in Split SUSY from the fact that we have light charginos and neutralinos with very heavy scalars, and the Higgs-boson mass is not constrained to be $m_{h^{0}}<140 \mathrm{GeV}$ when all superpartners are heavy, as in the MSSM. So, there is a new region containing allowed values for the Higgs-boson mass, $m_{H} \sim 140-153 \mathrm{GeV}$, which does not exist in the MSSM. On the other hand, the comparison of predictions of Split SUSY with the SM ones also leads to overlap regions between them. This region corresponds to a region with same values of the Higgs-boson mass in the two models. Since the region emerging from the Split SUSY predictions is larger than that
obtained from the SM, and by taking into account the experimental errors, the former might be slightly favoured (depending on the central experimental value). Even if we are concerning with just $M_{W}$ and $\sin ^{2} \theta_{\text {eff }}$ electroweak precision observables, this result could also be extracted from the analysis of radiative corrections to observables at LEP1 given in 16.

From now on, we focus on the differences between SM and Split SUSY predictions. To assess the importance of the Split SUSY contributions, we must compare these with the present and future experimental uncertainties and SM theoretical errors. The current experimental uncertainties are [46, 47]

$$
\begin{align*}
\Delta M_{W}^{\text {exp,today }} & \approx 34 \mathrm{MeV} \\
\Delta \sin ^{2} \theta_{\text {eff }}^{\text {exptoday }} & \approx 17 \times 10^{-5} ; \tag{3.1}
\end{align*}
$$

the expected experimental precision for the LHC is 48

$$
\begin{equation*}
\Delta M_{W}^{\mathrm{LHC}} \approx 15-20 \mathrm{MeV} ; \tag{3.2}
\end{equation*}
$$

and at a future linear collider running on the $Z$ peak and the $W W$ threshold (GigaZ) one expects 49-52, 20

$$
\begin{equation*}
\Delta M_{W}^{\exp , \text { future }} \approx 7 \mathrm{MeV}, \quad \Delta \sin ^{2} \theta_{\mathrm{eff}}^{\exp , f u t u r e} \approx 1.3 \times 10^{-5} \tag{3.3}
\end{equation*}
$$

On the other hand, the theoretical intrinsic uncertainties in the SM computation are [3]:

$$
\begin{array}{ll}
\Delta M_{W}^{\text {th,today, }, \mathrm{SM}} \approx 4 \mathrm{MeV}, & \Delta \sin ^{2} \theta_{\mathrm{eff}}^{\text {th,today, } \mathrm{SM}} \approx 5 \times 10^{-5}, \\
\Delta M_{W}^{\text {th,future,SM }} \approx 2 \mathrm{MeV}, & \Delta \sin ^{2} \theta_{\text {eff }}^{\text {th.future,SM }} \approx 2 \times 10^{-5} . \tag{3.4}
\end{array}
$$

We remark that the radiative corrections induced by Split SUSY in the $M_{W}$ and $\sin ^{2} \theta_{\text {eff }}$ precision observables and, in particular, their differences with respect to predictions from other models, depends strongly on the Higgs-boson mass. Besides, the role of the $\tan \beta$ parameter in this analysis is dictated by the fact that the Higgs-boson mass increases with $\tan \beta$ for small values of this parameter, around $1-5$. For larger values of $\tan \beta$, we found that the Higgs-boson mass remains stable. Figure 2 shows the result of the parameter scan in Split SUSY for the central experimental value of the top-quark mass $m_{t}=172.7 \mathrm{GeV}$, and two different values of $\tan \beta$. The results obtained when $\operatorname{taking} \tan \beta=1$ are displayed in the green/light-grey area of this figure. The black area represents $\tan \beta=10$. The Higgsboson mass runs between 114 GeV and 153 GeV , as predicted by Split SUSY. We can see that the effective leptonic weak mixing angle, $\sin ^{2} \theta_{\text {eff }}$, always decreases when $\tan \beta=10$ but, on the contrary, its value increases when $\tan \beta=1$ for some specific set of values of the other parameters, in particular when $\mu>0$ (see below). So, the correction to $\sin ^{2} \theta_{\text {eff }}$


Figure 3: The shifts $\Delta \sin ^{2} \theta_{\text {eff }}$ and $\Delta M_{W}$ in the $\left[M_{2}-\mu\right]$ plane for $m_{t}=172.7 \mathrm{GeV}$ and for $\tan \beta=1(\mathrm{a}, \mathrm{c})$ and $\tan \beta=10(\mathrm{~b}, \mathrm{~d})$. The shaded region corresponds to $m_{\chi}<100 \mathrm{GeV}$. Also shown is the line corresponding to a lightest chargino mass $m_{\chi}=250 \mathrm{GeV}$. The gluino mass is taken to be $M_{g}=500 \mathrm{GeV}$.
is positive for small values of $\tan \beta$ and $\mu>0$. The corrections to $M_{W}$ are positive over a large range of the parameter space. When $\tan \beta=1$ and $\mu>0$ we can also get negative corrections. We found that for values of $\tan \beta$ larger than 10 , the above conclusions remain unchanged.

In figure 3 we show the shifts $\Delta \sin ^{2} \theta_{\text {eff }}$ and $\Delta M_{W}$ in the $\left[M_{2}-\mu\right]$ plane. The shifts in the variables are defined as: $\Delta X \equiv X^{\text {Split SUSY }}-X^{\text {SM }}$, where the SM computation is performed using the Higgs-boson mass predicted by Split SUSY. The top-quark mass is fixed to its central value $m_{t}=172.7 \mathrm{GeV}$, while $\tan \beta=1$ in figures 3 a,c and $\tan \beta=10$ in figures 3 b b,d. The regions with a chargino mass smaller than 100 GeV are excluded. At the upper side of this figure we display the shifts on the effective leptonic weak mixing angle, $\Delta \sin ^{2} \theta_{\text {eff }}$ and, in the lower side the results for $\Delta M_{W}$. The Split-SUSY-induced shifts are
$\left|\Delta \sin ^{2} \theta_{\text {eff }}\right|<10 \times 10^{-5}$ and $\left|\Delta M_{W}\right|<20 \mathrm{MeV}$; as of today's data (3.1) they are smaller than the experimental error, and the data cannot discriminate between the SM and Split SUSY. The same conclusion applies to the accuracy reached at the LHC (3.2). However, the shifts are larger than the experimental accuracy of GigaZ (3.3) in certain regions of the parameter space. For $\tan \beta=1$, the shift in $\left|\Delta \sin ^{2} \theta_{\text {eff }}\right|$ is larger than $1.3 \times 10^{-5}$ for most of the explored region for $\mu>0$ and for the region with $\mu<0$ : $\mu \gtrsim-250 \mathrm{GeV}$ or $M_{2} \lesssim 150 \mathrm{GeV}$ (figure 3 a ). At $\tan \beta=10$ (figure 3 b ), $\left|\Delta \sin ^{2} \theta_{\text {eff }}\right|$ is larger than the future experimental accuracy (3.3) in a small region $M_{2} \lesssim 175-200 \mathrm{GeV}$ for $\mu>0$, and a large region $M_{2} \lesssim 200-500 \mathrm{GeV}$ for $\mu<0$. As far as $M_{W}$ is concerned, the LHC measurement (3.2) could only be useful in a small corner of the parameter space for $\mu<0$, $\tan \beta \gtrsim 10$. The GigaZ measurement (3.3) does not help for $\tan \beta=1, \mu>0$, owing to the cancellation of the corrections in the center of the region. For $\tan \beta=1, \mu<0$ there exists a small region for $M_{2} \lesssim 110 \mathrm{GeV}$ or $\mu>-110 \mathrm{GeV}$. For larger $\tan \beta$, the region of sensitivity is much larger. Summarizing the results of figure 3:

- Positive shifts of $\sin ^{2} \theta_{\text {eff }}$ are only possible at small $\tan \beta \simeq 1$ and $\mu>0$. They are large, and correlated with small and negative shifts of $M_{W}$. These large shifts are possible even for large values of the chargino masses $\left(m_{\chi}>250 \mathrm{GeV}\right)$.
- For $\tan \beta \simeq 1, \mu<0$ large negative shifts in $\sin ^{2} \theta_{\text {eff }}$ are possible, correlated with positive shifts in $M_{W}$, but $\sin ^{2} \theta_{\text {eff }}$ is the most sensitive of those observables.
- For large $\tan \beta \gtrsim 10$ and $\mu>0$, the sensitivity region is confined to small $M_{2} \lesssim 275-$ 375 GeV , with the largest shift provided by $\sin ^{2} \theta_{\text {eff }}$ for $\mu \gtrsim 300 \mathrm{GeV}$, and by $M_{W}$ otherwise.
- Finally, for large $\tan \beta \gtrsim 10$ and $\mu<0$, the largest sensitivity is provided by $\sin ^{2} \theta_{\text {eff }}$; it can reach GigaZ sensitivities even for moderate chargino masses ( $m_{\chi} \approx 250 \mathrm{GeV}$ ).

We would like to stress the fact that the results for negative $\mu$ are quite different from those of positive $\mu$. As figure 3 shows, changing the sign of $\mu$ can change the sign and the absolute value of the shifts significantly, so conclusions derived from an analysis of the $\mu>0$ scenario only do not necessarily apply to the complete Split SUSY parameter space.

To finish the discussion on the shifts $\Delta \sin ^{2} \theta_{\text {eff }}$ and $\Delta M_{W}$, the results of the difference between Split SUSY and SM predictions in the $M_{W}-\sin ^{2} \theta_{\text {eff }}$ plane are displayed in figure 4, together with the expected error ellipses of the future colliders (3.2) and (3.3) centered at the SM value. These variations for $\sin ^{2} \theta_{\text {eff }}$ and $M_{W}$ have to be compared with the numbers of eqs. (3.1)-(3.4). We can see that the shift $\Delta M_{W}$ can be up to 23 MeV at its maximum and, therefore, it is impossible to discriminate between models at the present experimental accuracy. However, future experiments could be probed with the future precision on $M_{W}$, if theoretical uncertainties will be sufficiently under control. On the other hand, the shifts $\Delta \sin ^{2} \theta_{\text {eff }}$ can easily reach values $\pm 2 \times 10^{-5}$, which is larger than both the expected experimental errors and the anticipated theoretical accuracies (3.4).

As a side note, we observe from figure 叩b that the current SM prediction of $M_{W}-\sin ^{2} \theta_{\text {eff }}$ would need a positive shift on both observables (together with a large value of $m_{t}$ ) to be closer to the central experimental value. Figures 3, $7^{4}$ show that the general trend of the Split SUSY contributions is a negative correlation of the shifts on both observables, that is, if $\Delta M_{W}>0$ then $\Delta \sin ^{2} \theta_{\text {eff }}<0$, an reciprocally. The region providing $\left(\Delta M_{W}>0, \Delta \sin ^{2} \theta_{\text {eff }}>0\right)$ is actually small and the largest region corresponds to $\left(\Delta M_{W}>0, \Delta \sin ^{2} \theta_{\text {eff }}<\right.$ $0)$ - c.f. figure 0 . Of course, since we are dealing with high precision observables, small deviations from the general trend are important, and refs. [12, (16] actually show that there are


Figure 4: Shifts of the differences between Split SUSY and SM predictions for $M_{W}$ and $\sin ^{2} \theta_{\text {eff }}$, scanning over the parameter space. Also shown are the ellipses for the prospective accuracies at LHC/ILC (large ellipse) and GigaZ (small ellipse). points of the parameter Split SUSY space that fit better than the SM the experimental value of the electroweak precision observables.

## 4. Conclusions

In conclusion, we have computed the Split SUSY contributions to the electroweak precision observables $M_{W}$ and $\sin ^{2} \theta_{\text {eff }}$ arising from a heavy scalar spectrum and light charginos and neutralinos. For the computation, we have evaluated the Higgs-boson mass by using the renormalization-group evolution equations, the $\Delta r$-parameter, and then $M_{W}$ and $\sin ^{2} \theta_{\text {eff }}$. Numerically we compared the effects of radiative corrections to these observables induced by Split SUSY, SM and the MSSM, and with present and future experimental and theoretical accuracies. We find that the shifts induced in Split SUSY models are smaller than present experimental accuracies (3.1), and therefore no conclusion can be drawn with respect to the validity of this model. With the anticipated LHC accuracy on $M_{W}$, a small corner of the parameter space can be explored. However, only with the GigaZ option of the ILC the experiment would be sensitive to the Split SUSY corrections to these observables. In this option, the effective leptonic mixing angle $\left(\sin ^{2} \theta_{\text {eff }}\right)$ is the most sensitive of the two observables. For moderate and large $\tan \beta$, the lightest chargino must be relatively light, $m_{\chi} \lesssim 250 \mathrm{GeV}$, which should have already been detected either at the LHC or the ILC before the GigaZ era. The observables provide, however, a high-precision test of the model. An interesting case is a scenario with low $\tan \beta \simeq 1$ and positive $\mu$, where large shifts in $\sin ^{2} \theta_{\text {eff }}$ are expected, even for large values of the chargino masses.

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